7.11 Summary

Fact.

Any automaton $\mathcal{A}$ is a computer that processes its input $x \in \Sigma^*$ in time $|x|$ and using constant space (i.e. the number of states) regardless of the length of the input (i.e. the time it takes to process the input).

This is a fact based on Definitions 2.3.1 and 3.3.1.

Fact.

The left abstract automaton of a language $L \in \Sigma^*$ is isomorphic to the right abstract automaton of the reversed language $L^R \in \Sigma^*$.

See Section 7.6 and Exercises 7.12.5 and 20.3.10.

Fallacy.

Symmetry of left and right theories imply that the number of states of the left minimal automaton is equal to the number of states of the right minimal automaton of any language $L \in \Sigma^*$!

This is definitely a wrong reasoning. See Exercise 7.12.7 for a counterexample and also Sections 7.6 and 7.7 for more details. Also, Examples 7.7.5 and 4.7.6 show that not only this *may* happen sometimes, but also the automata *may* even be isomorphic.

Pitfall.

An abstract automaton is a computer!

This is definitely a wrong statement. An abstract automaton (see Definition 1.7.5) is a *computational model* as defined in Definition **??**, while an *automaton* is a *computer* as defined in Definition 3.3.1 (also see Definition 2.3.1). Within this context, a *preautomaton* is an *algebraic structure* (see Sections 7.2 and 20.1.6).

7.12 Exercises

7.12.1. Answer the following question and provide your complete analysis of the cases.

a) Characterize all computers (according to Definition 2.3.1) that operate in real time and use constant memory.

b) Try to characterize some interesting subclasses of regular languages. Why this is not an easy task?

7.12.2. Show that given two preautomata $\mathcal{A}$ and $\mathcal{A}$, if $\sigma : \mathcal{A} \bullet\!\!\longrightarrow \mathcal{B}$ is a one to one and onto homomorphism, then $\sigma^{-1} : \mathcal{B} \bullet\!\!\longrightarrow \mathcal{A}$ is also a homomorphism.

7.12.3. Prove that a preautomaton $\mathcal{A}(Q, q_0 \in Q, \{\tau_i : Q \bullet\!\!\longrightarrow Q\}_{i\in\Sigma})$ is reduced if each state $q \in Q$ is accessible from q_0.

7.12.4. Show that if $\sigma : \mathcal{A} \bullet\!\!\longrightarrow \mathcal{B}$ is a homomorphism of preautomata and $\mathcal{B}$ is reduced, the σ is an onto map.

7.12.5. Explicitly describe the left congruence relation corresponding to the left canonical map for the $\{0, 1\}^*$-preautomaton of Example 6.7.4.

7.12.6. Let $\simeq_A$ and $\simeq_B$ be the (right) congruence relations corresponding to two given preautomata, $\mathcal{A}$ and $\mathcal{B}$, respectively. Explicitly, describe the preautomata corresponding to the following congruence relations.

- $(\simeq_A \cap \simeq_B)$.
- The closure of $(\simeq_A \cup \simeq_B)$, i.e. the smallest congruence relation containing the equivalence relation $(\simeq_A \cup \simeq_B)$ (why does this congruence relation exist? see Section 20.1.7).

7.12.7. Given a language $L \in \Sigma^*$, by comparing the partitions corresponding to the equivalence relations $\simeq^r_L$ and $\simeq^l_L$ prove that although the number of states of minimal automata $\mathcal{M}_r(L)$ and $\mathcal{M}_l(L)$ may be different, but either both L and L^R are regular languages or both of these languages are not regular. Note that this fact shows that there is no ambiguity in Definition 6.7.5.